Find $\frac{dy}{dx}$ algebraically. Check your answers on calc.

(a) $y = (3x^2 - x)^3$

$3(3x^2-x)^2 \cdot (6x-1)$

$(18x-3)(3x^2-x)^2$

(b) $y = x\sqrt{1-x^2}$

$$\frac{-2x^2+1}{\sqrt{1-x^2}}$$

(c) $y = x\sin^2(\pi x - 2)$

$2\pi x \sin(\pi x-2)\cos(\pi x-2) + \sin^2(\pi x-2)$

(d) $y = \sqrt{\frac{2x}{x^2+2}}$

$$\frac{-x^2+2}{\sqrt{(x^2+3)^3}\,\sqrt{2x}}$$

$$y = (3x^2 - x)^3 \qquad \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$y = u^3 \qquad \frac{dy}{du} = 3u^2$$

$$u = 3x^2 - x \qquad \frac{du}{dx} = 6x - 1$$

$$\frac{dy}{dx} = 3(3x^2 - x)^2 \cdot (6x - 1)$$

$y = x\sqrt{1-x^2}$

$y = \sqrt{1-x^2}$

$y = \sqrt{u}$, $u^{\frac{1}{2}}$ $\quad \frac{dy}{du} = \frac{1}{2}\cdot u^{-\frac{1}{2}} \longrightarrow \frac{1}{2\sqrt{u}}$

$u = 1-x^2 \quad \frac{du}{dx} = -2x$

$\frac{dy}{dx} = \frac{1}{\cancel{2}\sqrt{1-x^2}} \cdot -\cancel{2}x$

$\frac{dy}{dx} = -\frac{x}{\sqrt{1-x^2}}$

$$\frac{dy}{dx} = x\left(-\frac{x}{\sqrt{1-x^2}}\right) + \sqrt{1-x^2}\,(1)$$

$$= -\frac{x^2}{\sqrt{1-x^2}} + \frac{\sqrt{1-x^2}}{1}\cdot\frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}$$

$$= \frac{-x^2}{\sqrt{1-x^2}} + \frac{1-x^2}{\sqrt{1-x^2}}$$

$$= \frac{-2x^2+1}{\sqrt{1-x^2}}$$

$$y = \sin^2(\pi x - 2) \Rightarrow \left(\sin(\pi x - 2)\right)^2$$

$$y = u^2 \qquad \frac{dy}{du} = 2u$$

$$u = \sin(\pi x - 2) \longrightarrow \frac{du}{dx} = \cos(\pi x - 2) \cdot \pi$$

$$u' = \sin(g) \qquad \frac{du'}{dg} = \cos(g)$$

$$g = \pi x - 2 \qquad \frac{dg}{dx} = \pi$$

$$\frac{dy}{dx} = 2\pi \sin(\pi x - 2)\cos(\pi x - 2)$$

$$y = x \sin^2(\pi x - 2)$$

$$\frac{dy}{dx} = x \cdot \left[2\pi \sin(\pi x - 2)\cos(\pi x - 2)\right] + \sin^2(\pi x - 2)(1)$$

$$\boxed{= 2\pi x \sin(\pi x - 2)\cos(\pi x - 2) + \sin^2(\pi x - 2)}$$

$$y = \sqrt{\frac{2x}{x^2+2}}$$

$$y = \sqrt{u}, \quad u^{\frac{1}{2}} \qquad \frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}} \rightarrow \frac{1}{2\sqrt{u}}$$

$$u = \frac{2x}{x^2+2} \qquad \frac{du}{dx} = \frac{(x^2+2)(2) - (2x)(2x)}{(x^2+2)^2}$$

$$= \frac{2x^2+4-4x^2}{(x^2+2)^2} = \frac{-2x^2+4}{(x^2+2)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{2\sqrt{\frac{2x}{x^2+2}}} \cdot \frac{-2x^2+4}{(x^2+2)^2}}$$

$$= \frac{1}{2\frac{\sqrt{2x}}{\sqrt{x^2+2}}}$$

$$= \frac{\sqrt{x^2+2}}{\cancel{2}\sqrt{2x}} \cdot \frac{\cancel{2}(-x^2+2)}{(x^2+2)^2} = \frac{\cancel{(x^2+2)^{\frac{1}{2}}}}{\sqrt{2x}} \cdot \frac{(-x^2+2)}{(x^2+2)^{\cancel{4}/2}}$$

$$= \frac{-x^2+2}{\sqrt{2x} \cdot \sqrt{(x^2+2)^3}}$$

3. $-\sqrt{3}\sin(\sqrt{3}x)$ 4. $(2-3x^2)\sec^2(2x-x^3)$ 6. $\frac{10}{x^2}\csc^2\left(\frac{2}{x}\right)$ 9. $3\sin\left(\frac{\pi}{2}-3t\right)$ 13. $-2(x+\sqrt{x})^{-3}\left(1+\frac{1}{2\sqrt{x}}\right)$ 16. $8x^3(2x-5)^3+3x^2(2x-5)^4$ $=x^2(2x-5)^3(14x-15)$

Section 3.6 Exercises

In Exercises 1–8, use the given substitution and the Chain Rule to find dy/dx.

1. $y = \sin(3x+1)$, $u = 3x+1$ $3\cos(3x+1)$
2. $y = \sin(7-5x)$, $u = 7-5x$ $-5\cos(7-5x)$
3. $y = \cos(\sqrt{3}x)$, $u = \sqrt{3}x$
4. $y = \tan(2x-x^3)$, $u = 2x-x^3$
5. $y = \left(\dfrac{\sin x}{1+\cos x}\right)^2$, $u = \dfrac{\sin x}{1+\cos x}$ $\dfrac{2\sin x}{(1+\cos x)^2}$
6. $y = 5\cot\left(\dfrac{2}{x}\right)$, $u = \dfrac{2}{x}$
7. $y = \cos(\sin x)$, $u = \sin x$ $-\sin(\sin x)\cos x$
8. $y = \sec(\tan x)$, $u = \tan x$ $\sec(\tan x)\tan(\tan x)\sec^2 x$

In Exercises 9–12, an object moves along the x-axis so that its position at any time $t \geq 0$ is given by $x(t) = s(t)$. Find the velocity of the object as a function of t.

9. $s = \cos\left(\dfrac{\pi}{2} - 3t\right)$
10. $s = t\cos(\pi - 4t)$ $4t\sin(\pi-4t)+\cos(\pi-4t)$
11. $s = \dfrac{4}{3\pi}\sin 3t + \dfrac{4}{5\pi}\cos 5t$ $\dfrac{4}{\pi}\cos 3t - \dfrac{4}{\pi}\sin 5t$
12. $s = \sin\left(\dfrac{3\pi}{2}t\right) + \cos\left(\dfrac{7\pi}{4}t\right)$ $\dfrac{3\pi}{2}\cos\dfrac{3\pi t}{2} - \dfrac{7\pi}{4}\sin\dfrac{7\pi t}{4}$

In Exercises 13–24, find dy/dx. If you are unsure of your answer, use

In Exercises 33–38, find the value of $(f \circ g)'$ at the given value of x.

33. $f(u) = u^5+1$, $u = g(x) = \sqrt{x}$, $x = 1$ $5/2$
34. $f(u) = 1 - \dfrac{1}{u}$, $u = g(x) = \dfrac{1}{1-x}$, $x = -1$ 1
35. $f(u) = \cot\dfrac{\pi u}{10}$, $u = g(x) = 5\sqrt{x}$, $x = 1$ $-\pi/4$
36. $f(u) = u + \dfrac{1}{\cos^2 u}$, $u = g(x) = \pi x$, $x = \dfrac{1}{4}$ 5π
37. $f(u) = \dfrac{2u}{u^2+1}$, $u = g(x) = 10x^2+x+1$, $x = 0$ 0
38. $f(u) = \left(\dfrac{u-1}{u+1}\right)^2$, $u = g(x) = \dfrac{1}{x^2} - 1$, $x = -1$ -8

What happens if you can write a function as a composite in different ways? Do you get the same derivative each time? The Chain Rule says you should. Try it with the functions in Exercises 39 and 40.

39. Find dy/dx if $y = \cos(6x+2)$ by writing y as a composite with
    - **(a)** $y = \cos u$ and $u = 6x+2$. $-6\sin(6x+2)$
    - **(b)** $y = \cos 2u$ and $u = 3x+1$. $-6\sin(6x+2)$

$4t \sin(\pi - 4t) + \cos(\pi - 4t)$

9. $s = \cos\left(\dfrac{\pi}{2} - 3t\right)$

10. $s = t\cos(\pi - 4t)$

11. $s = \dfrac{4}{3\pi}\sin 3t + \dfrac{4}{5\pi}\cos 5t$ $\quad \dfrac{4}{\pi}\cos 3t - \dfrac{4}{\pi}\sin 5t$

12. $s = \sin\left(\dfrac{3\pi}{2}t\right) + \cos\left(\dfrac{7\pi}{4}t\right)$ $\quad \dfrac{3\pi}{2}\cos\dfrac{3\pi t}{2} - \dfrac{7\pi}{4}\sin\dfrac{7\pi t}{4}$

In Exercises 13–24, find dy/dx. If you are unsure of your answer, use NDER to support your computation.

13. $y = (x + \sqrt{x})^{-2}$

14. $y = (\csc x + \cot x)^{-1}$ $\quad \dfrac{\csc x}{\csc x + \cot x}$

15. $y = \sin^{-5} x - \cos^3 x$

16. $y = x^3(2x - 5)^4$

17. $y = \sin^3 x \tan 4x$

18. $y = 4\sqrt{\sec x + \tan x}$

19. $y = \dfrac{3}{\sqrt{2x + 1}}$ $\quad -3(2x + 1)^{-3/2}$

20. $y = \dfrac{x}{\sqrt{1 + x^2}}$ $\quad (1 + x^2)^{-3/2}$

21. $y = \sin^2(3x - 2)$

22. $y = (1 + \cos 2x)^2$

23. $y = (1 + \cos^2 7x)^3$

24. $y = \sqrt{\tan 5x}$ $\quad \dfrac{5}{2}(\tan 5x)^{-1/2}\sec^2 5x$

In Exercises 25–28 find $dr/d\theta$.

25. $r = \tan(2 - \theta)$ $\quad -\sec^2(2 - \theta)$

26. $r = \sec 2\theta \tan 2\theta$

27. $r = \sqrt{\theta \sin \theta}$ $\quad \dfrac{\theta\cos\theta + \sin\theta}{2\sqrt{\theta\sin\theta}}$

28. $r = 2\theta\sqrt{\sec\theta}$ $\quad \sqrt{\sec\theta}\,(\theta\tan\theta + 2)$

In Exercises 29–32, find y''.

29. $y = \tan x$ $\quad 2\sec^2 x \tan x$

30. $y = \cot x$ $\quad 2\csc^2 x \cot x$

31. $y = \cot(3x - 1)$

32. $y = 9\tan(x/3)$ $\quad 2\sec^2\dfrac{x}{3}\tan\dfrac{x}{3}$

15. $-5\sin^{-6} x\cos x + 3\cos^2 x \sin x$

18. $2\sec x\sqrt{\sec x + \tan x}$

17. $4\sin^3 x\sec^2 4x + 3\sin^2 x\cos x\tan 4x$

21. $6\sin(3x - 2)\cos(3x - 2) = 3\sin(6x - 4)$

22. $-4(1 + \cos 2x)\sin 2x$

26. $2\sec^3 2\theta + 2\sec 2\theta\tan^2 2\theta$

23. $-42(1 + \cos^2 7x)^2\cos 7x\sin 7x$

31. $18\csc^2(3x - 1)\cot(3x - 1)$

What ha
ways? I
says yo

39. Fin
wit

(a)

(b)

40. Fin
wit

(a)

(b)

In Exerc
at the po

41. $x =$

42. $x =$

43. $x =$

44. $x =$

45. $x =$

46. $x =$

47. $x =$

48. $x =$

10:2

Sect. 3.6 1-39